Exercise 66

(a) Use a graph to estimate the absolute maximum and minimum values of the function to two decimal places.

(b) Use calculus to find the exact maximum and minimum values.

$$f(x) = e^x + e^{-2x},\ 0 \le x \le 1$$

Solution

Take the derivative of the function.

$$\begin{aligned}
f'(x) &= \frac{d}{dx}(e^x + e^{-2x}) \\
&= \frac{d}{dx}(e^x) + \frac{d}{dx}(e^{-2x}) \\
&= e^x + (e^{-2x}) \cdot \frac{d}{dx}(-2x) \\
&= e^x + (e^{-2x}) \cdot (-2) \\
&= e^x - 2e^{-2x}
\end{aligned}$$

Set $f'(x) = 0$ and solve for x.

$$\begin{aligned}
e^x - 2e^{-2x} &= 0 \\
e^x &= 2e^{-2x} \\
e^{2x} \times e^x &= 2e^{-2x} \times e^{2x} \\
e^{3x} &= 2 \\
\ln e^{3x} &= \ln 2 \\
3x &= \ln 2 \\
x &= \frac{1}{3}\ln 2 \approx 0.231049
\end{aligned}$$

$x = 0.231049$ is within $0 \le x \le 1$, so evaluate f at this value.

$$f\left(\frac{1}{3}\ln 2\right) = e^{\frac{1}{3}\ln 2} + e^{-\frac{2}{3}\ln 2} = e^{\ln 2^{1/3}} + e^{\ln 2^{-2/3}} = 2^{1/3} + 2^{-2/3} \approx 1.88988 \quad \text{(absolute minimum)}$$

Now evaluate the function at the endpoints. The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $0 \leq x \leq 1$.

$$f(0) = e^0 + e^{-2(0)} = 2$$

$$f(1) = e^1 + e^{-2(1)} = e + \frac{1}{e^2} \approx 2.85362 \qquad \text{(absolute maximum)}$$

The graph below illustrates these results.